



Subject: Grade 11 Mathematics

Topic: Mathematics: Term 1 - Mini-Exam

Total: 111 Marks

1. false

2 marks

Explanation:

To get from $x^2 + bx + c = 0$ to $(x + p)^2 + q = 0$, completing the square is done.

Complete the square with $x^2 - 5x - 4 = 0$ to convert it to the form $(x + p)^2 + q = 0$.

To do this, determine a constant value of $(\frac{b}{2})^2$, then add and subtract this constant value.

The b -value in $x^2 - 5x - 4 = 0$ is -5:

$$(\frac{-5}{2})^2 = \frac{25}{4}$$

$\frac{25}{4}$ is the constant value that needs to be added and subtracted:

$$x^2 - 5x + \frac{25}{4} - \frac{25}{4} - 4 = 0$$

The constant is added and then subtracted so that the value of the expression remains unchanged.

The first three terms form a **perfect square trinomial** (a trinomial that has identical factors):

$$x^2 - 5x + \frac{25}{4} = (x - \frac{5}{2})(x - \frac{5}{2}) = (x - \frac{5}{2})^2$$

$$\therefore x^2 - 5x + \frac{25}{4} - \frac{25}{4} - 4 = 0$$

$$(x - \frac{5}{2})^2 - \frac{25}{4} - 4 = 0$$

Simplify the constant values

$$(x - \frac{5}{2})^2 - \frac{41}{4} = 0$$



2. true

2 marks

Explanation:

$$x + 2y = 0 \quad \text{equation 1}$$

$$y - xy - \frac{1}{2}x^2 = -2 \quad \text{equation 2}$$

Rearrange equation 1:

$$x = -2y \quad \text{equation 3}$$

Substitute equation 3 into equation 2:

$$y - (-2y)(y) - \frac{1}{2}(-2y)^2 = -2$$

$$y + 2y^2 - \frac{1}{2}(4y^2) = -2$$

$$y + 2y^2 - 2y^2 = -2$$

$$y = -2$$

Substitute $y = -2$ into equation 3:

$$x = -2(-2)$$

$$x = 4$$

3. $-3/2$ | $-1,5$ | $3/-2$ | $x=-3/2$ | $x=-1,5$ | $x=3/-2$

3 marks

Explanation:

Given:

$$\frac{x-3}{x^2+4x+3} = \frac{2x}{x+1}$$

Factorise the denominators where possible:

$$\frac{x-3}{(x+3)(x+1)} = \frac{2x}{x+1}$$

Multiply each term with the *LCD* of $(x+3)(x+1)$:

$$\frac{(x-3)(x+3)(x+1)}{(x+3)(x+1)} = \frac{2x(x+3)(x+1)}{x+1}$$

Simplify by **cancelling factors**:

$$x-3 = 2x(x+3)$$

Solve for x :

$$\begin{aligned} x-3 &= 2x^2+6x \\ 2x^2+5x+3 &= 0 \\ (2x+3)(x+1) &= 0 \end{aligned}$$

$$\therefore x = -\frac{3}{2} \text{ or } x = -1$$

 $x \neq -1$ as this will result in a denominator of 0 which will have an undefined answer.The **only valid solution** is $x = -\frac{3}{2}$.

4. true

2 marks

Explanation: $(\theta - 180^\circ)$ is treated as a negative angle (an angle in the clockwise direction). To convert it to an angle in the anti-clockwise direction, add 360° .

$$\begin{aligned} &\sin(\theta - 180^\circ + 360^\circ) \\ &= \sin(\theta + 180^\circ) \\ &= \sin(180^\circ + \theta) \end{aligned}$$

This is a reduction formula in quadrant III:

$$\therefore -\sin \theta$$

5. $45,2 \mid 45.2 \mid 45,2 \text{ degrees} \mid 45.2 \text{ degrees} \mid 45,2^\circ \mid 45.2^\circ$

3 marks

Explanation:

$$2 \sin 3\theta = 1,4 \quad \text{Divide by 2}$$

$$\sin 3\theta = 0,7$$

Solve for 3θ : $3\theta = \sin^{-1}(0,7) = 44,427\dots^\circ$

The general solution for $\sin \theta = c$ is:

$$\theta = \sin^{-1} c \text{ or } \theta = 180^\circ - \sin^{-1} c.$$

We use these two reduction formulae because $\sin \theta$ will be positive in these quadrants.

$$3\theta = 44,427\dots^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$$

$$\theta = 14,8^\circ + k \cdot 120^\circ$$

or

$$3\theta = 180^\circ - 44,427\dots^\circ + k \cdot 360^\circ$$

$$\theta = 45,2^\circ + k \cdot 120^\circ; k \in \mathbb{Z}$$

Remember that when the equation is divided, to divide the angle and the period.

The solution needs to be in the interval $\theta \in [45^\circ; 90^\circ]$.

$$\therefore \theta = 45,2^\circ$$

6. $-4 \mid x = -4$

3 marks

Explanation:

Isolate the surd:

$$\sqrt{5-x} + 1 + x = 0$$

$$\sqrt{5-x} = -1 - x$$

Square both sides:

$$(\sqrt{5-x})^2 = (-1-x)^2$$

$$5-x = x^2 + 2x + 1$$

$$0 = x^2 + 3x - 4$$

Factorise and equate each factor to 0:

$$(x+4)(x-1) = 0$$

$$x+4 = 0 \text{ or } x-1 = 0$$

$$x = -4 \quad \text{or} \quad x = 1$$

Substitute to find relevant answer(s):

$$\text{LHS: } \sqrt{5-(-4)} + 1 + (-4) = \sqrt{9} - 3 = 0 \quad \text{RHS: } 0$$

$$\text{LHS: } \sqrt{5-(1)} + 1 + (1) = \sqrt{4} + 2 = 4 \quad \text{RHS: } 0$$

$\therefore x = -4$ is the only relevant answer.

7. B: $\sin 65^\circ$

4 marks

Explanation:

245° is in quadrant III. The reduction formula for quadrant III is $(180^\circ + \theta)$.

$$\therefore 245^\circ = 180^\circ + 65^\circ$$

130° is in quadrant II.

The reduction formula for quadrant II is $(180^\circ - \theta)$.

$$\therefore 130^\circ = 180^\circ - 50^\circ$$

310° is in quadrant IV.

The reduction formula for quadrant IV is $(360^\circ - \theta)$.

$$\therefore 310^\circ = 360^\circ - 50^\circ$$

$\sin \theta < 0$ in quadrant III.

$\cos \theta < 0$ in quadrant II.

$\cos \theta > 0$ in quadrant IV.

$$\begin{aligned} & \frac{\sin 245^\circ \cdot \cos 130^\circ}{\cos 310^\circ} \\ = & \frac{\sin(180^\circ + 65^\circ) \cdot \cos(180^\circ - 50^\circ)}{\cos(360^\circ - 50^\circ)} \\ = & \frac{-\sin 65^\circ \cdot -\cos 50^\circ}{\cos 50^\circ} && \text{Simplify} \\ = & \sin 65^\circ \end{aligned}$$

8. C: $-\frac{12}{35}$

4 marks

Explanation:

$$\frac{\sin(360^\circ - \theta)}{\cos(180^\circ - \theta)}$$

$$= \frac{-\sin \theta}{-\cos \theta}$$

Apply reduction formulae

$$= \frac{-\frac{12}{37}}{-\frac{35}{37}}$$

Substitute

$$= \frac{12}{37} \div \frac{-35}{37}$$

Divide fractions

$$= \frac{12}{37} \times \frac{37}{-35}$$

Simplify

$$= -\frac{12}{35}$$

9. A: $-\sqrt{1-p^2}$

4 marks

Explanation:

$$\sin 25^\circ = \frac{p}{1} = \frac{y}{r}$$

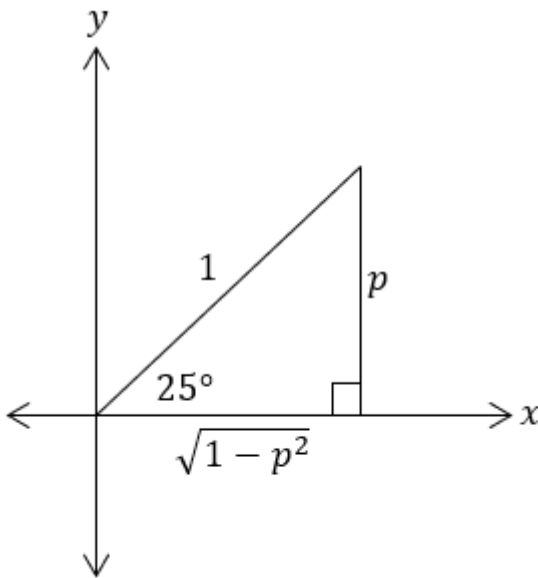
Use Pythagoras to determine x .

$$x^2 = r^2 - y^2$$

$$x^2 = 1^2 - p^2$$

$$x = \sqrt{1 - p^2}$$

The right-angled triangle is as follows:



Express $\cos 205^\circ$ in terms of 25° with the use of a reduction formula:

$$\cos 205^\circ = \cos(180^\circ + 25^\circ) \quad \text{Reduce}$$

$$= -\cos 25^\circ$$

$$= -\frac{x}{r}$$

$$= -\frac{\sqrt{1 - p^2}}{1}$$

$$= -\sqrt{1 - p^2}$$

10. B: $\beta = -33,69^\circ + k.180^\circ; k \in \mathbb{Z}$

4 marks

Explanation:

$$3 \sin \beta + 2 \cos \beta = 0$$

Divide the equation by $\cos \beta$.

$$\frac{3 \sin \beta}{\cos \beta} + \frac{2 \cos \beta}{\cos \beta} = \frac{0}{\cos \beta}$$

Simplify.

$$3 \tan \beta + 2 = 0$$

Solve the equation involving $\tan \beta$.

$$\tan \beta = -\frac{2}{3}$$

$$\beta = \tan^{-1}\left(-\frac{2}{3}\right)$$

$$\beta = -33,69^\circ + k.180^\circ; k \in \mathbb{Z}$$

11. false

2 marks

Explanation:

$$\frac{pq^{-1} - p^{-1}q}{q^{-1} - p^{-1}}$$

Change the negative exponents to positive exponents:

$$= \frac{\frac{p}{q} - \frac{q}{p}}{\frac{1}{q} - \frac{1}{p}}$$

Make a common denominator in both numerator and denominator:

$$= \frac{\frac{p^2 - q^2}{pq}}{\frac{p - q}{pq}}$$

Divide with "tip and times":

$$= \frac{p^2 - q^2}{pq} \times \frac{pq}{p - q}$$

Factorise the difference of two squares:

$$= \frac{(p + q)(p - q)}{pq} \times \frac{pq}{p - q}$$

Cancel factors:

$$= p + q$$

12. 2,78 | 0,72

4 marks

Explanation:

$$4 - 7x = -2x^2$$

$$2x^2 - 7x + 4 = 0$$

Rewrite in **standard form**

This cannot be factorised and can be solved using the **quadratic formula**.

$$a = 2, b = -7 \text{ and } c = 4$$

Substitute into the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$$

$$\therefore x = 0,72 \text{ or } x = 2,78$$

13. -2 | 3 | 0 | 5

8 marks

Explanation:

Write down both equations.

$$y - 3 + x = 0 \quad \text{Equation 1}$$

$$x^2 - y = 2x + 3 \quad \text{Equation 2}$$

Manipulate equation 1 to form:

$$y = 3 - x \quad \text{Equation 3}$$

Substitute this newly formed equation 3 into the equation you have not used, in this case, equation 2.

$$x^2 - (3 - x) = 2x + 3$$

$$x^2 - 3 + x = 2x + 3$$

Rewrite in standard form and solve the quadratic equation.

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$\therefore x = 3 \text{ or } x = -2.$$

If $x = 3$ then,

$$y = 3 - 3 = 0$$

If $x = -2$ then,

$$y = 3 - (-2) = 5$$

14. A: $x = 4$ or $x = -3$

4 marks

Explanation:

$$x^2 - x - 12 = 0$$

$$(x - 4)(x + 3) = 0 \quad \text{Factorise the trinomial}$$

$$x - 4 = 0 \text{ or } x + 3 = 0 \quad \text{Set each factor equal to zero}$$

$$\therefore x = 4 \text{ or } x = -3$$

15. true

2 marks

Explanation:

$$7 \cdot 16^x = 224$$

Rewrite the bases of both sides as a product of its prime factors.

$$7 \cdot 2^{4x} = 7 \cdot 2^5$$

This is therefore a true statement.

Explanation:

Subtract 6 on both sides to **isolate the surd** and **square on both sides of the equation**.

$$6 - \sqrt{x+2} = x + 2$$

$$-\sqrt{x+2} = x - 4$$

$$(-\sqrt{x+2})^2 = (x-4)^2$$

$$x + 2 = x^2 - 8x + 16$$

Set the equation equal to 0 and then **factorise** the trinomial.

$$0 = x^2 - 9x + 14$$

$$0 = (x-7)(x-2)$$

$$\therefore x = 7 \text{ or } x = 2$$

Test both solutions:

If $x = 7$:

$$\begin{array}{ll} \text{LHS: } 6 - \sqrt{7+2} = 3 & \text{RHS: } 7 + 2 = 9 \\ & 3 \neq 9 \end{array}$$

LHS $\neq$ RHS, so $x = 7$ is **not a valid solution**.

If $x = 2$:

$$\begin{array}{ll} \text{LHS: } 6 - \sqrt{2+2} = 4 & \text{RHS: } 2 + 2 = 4 \\ & 4 = 4 \end{array}$$

LHS=RHS, so $x = 2$ is a **valid solution**.

Explanation:

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin \theta \cos \theta} = 1 - \tan \theta$$

On the left-hand side, the numerator can be **factorised** using the **difference of squares** and the denominator can be factorised using **highest common factor**:

$$\therefore \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)}{\cos \theta(\cos \theta + \sin \theta)}$$

The left-hand side can be simplified to (cancel the same sets of brackets):

$$\begin{aligned} & \frac{\cos \alpha - \sin \alpha}{\cos \alpha} \\ &= \frac{\cos \alpha}{\cos \alpha} - \frac{\sin \alpha}{\cos \alpha} \end{aligned}$$

The first term is equal to 1 and the second term is $\tan \theta$.

Explanation:

Left-hand side:

$$\frac{3 - 3 \sin \theta}{\sin \theta - \sin^2 \theta}$$

Take out a common factor in the numerator and denominator:

$$= \frac{3(1 - \sin \theta)}{\sin \theta(1 - \sin \theta)}$$

Cancel out $1 - \sin \theta$:

$$= \frac{3}{\sin \theta}$$

$$\therefore B = 3$$

19. -1

3 marks

Explanation:

For co-functions, the sine and cosine functions swap. The sign of the original function applies to its co-function in that particular quadrant.

In quadrant I, all functions are positive.

$$\cos(90^\circ - \theta) = \sin \theta$$

In quadrant II, $\cos \theta$ would be negative, so the co-function remains negative.

$$\cos(90^\circ + \theta) = -\sin \theta$$

$$\frac{\cos(90^\circ - \theta) \cdot \cos(90^\circ + \theta)}{\sin(180^\circ + \theta) \cdot \sin(360^\circ - \theta)}$$

$$= \frac{\sin \theta \cdot -\sin \theta}{-\sin \theta \cdot -\sin \theta} \quad \text{Simplify}$$

$$= -1$$

20. false

2 marks

Explanation:

Let $x^2 + 8x = k$:

$$k - 1 + \frac{13}{k + 4} = 9$$

Multiply by $(k + 4)$:

$$(k - 1)(k + 4) + 13 = 9(k + 4)$$

Simplify:

$$(k - 1)(k + 4) + 13 = 9k + 36$$

This equation can further be simplified:

$$k^2 + 3k - 4 + 13 = 9k + 36$$

$$k^2 - 6k - 27 = 0$$

Solve for k and then for x .

21. C: $\frac{2}{3}$

4 marks

Explanation:

$$\frac{8^x \cdot 6^{x-3} \cdot 9^{1-x}}{16^{x-1} \cdot 3^{-x}} \quad \text{Change bases to prime}$$

$$= \frac{(2^4)^x \cdot (2 \cdot 3)^{x-3} \cdot (3^2)^{1-x}}{(2^4)^{x-1} \cdot 3^{-x}}$$

Apply exponent law 3 to get rid of brackets:

$$= \frac{2^{4x} \cdot 2^{x-3} \cdot 3^{x-3} \cdot 3^{2-2x}}{2^{4x-4} \cdot 3^{-x}}$$

Apply Law 1 to same bases in numerator and denominator:

$$= \frac{2^{4x-3} \cdot 3^{-x-1}}{2^{4x-4} \cdot 3^{-x}}$$

Apply Law 2 to same bases:

$$= 2^{4x-3-(4x-4)} \cdot 3^{-x-1-(-x)}$$

$$= 2^1 \cdot 3^{-1}$$

$$= \frac{2}{3}$$

22. $p \leq 2 \mid p = 2$

4 marks

Explanation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{8 - 4p}}{6}$$

$$\therefore b^2 - 4ac = 8 - 4p$$

For real roots:

$$b^2 - 4ac \geq 0$$

$$8 - 4p \geq 0$$

$$-4p \geq -8$$

$$\therefore p \leq 2$$

For equal roots:

$$b^2 - 4ac = 0$$

$$8 - 4p = 0$$

$$-4p = -8$$

$$\therefore p = 2$$

23. A: 3^x

4 marks

Explanation:

$$\frac{3^{2x} - 3^x}{3^x - 1} \quad \text{Expand } 3^{2x}$$

$$= \frac{3^{x+x} - 3^x}{3^x - 1}$$

$$= \frac{3^x \cdot 3^x - 3^x}{3^x - 1}$$

Take out a common factor of 3^x in numerator:

$$= \frac{3^x(3^x - 1)}{3^x - 1}$$

Simplify:

$$= 3^x$$

Explanation:

Rewrite the equation in **standard form**:

$$x^2 + m + 3 = mx$$

$$x^2 - mx + (m + 3) = 0$$

For **equal roots** $\Delta = 0$

$$a = 1, b = -m \text{ and } c = m + 3.$$

Substitute the values into Δ .

$$b^2 - 4ac = 0$$

$$(-m)^2 - 4(1)(m + 3) = 0$$

$$m^2 - 4m - 12 = 0$$

$$(m - 6)(m + 2) = 0$$

$$\therefore m = 6 \text{ or } m = -2$$

Explanation:

$$2 \cdot 3^x + 3^{x+1} = 15 \quad \text{Expand}$$

$$2 \cdot 3^x + 3^x \cdot 3^1 = 15 \quad \text{Take out a common factor}$$

$$3^x (2 + 3) = 15 \quad \text{Simplify bracket}$$

$$3^x (5) = 15$$

$$3^x = \frac{15}{5}$$

This can now be solved:

$$3^x = 3$$

$$\therefore x = 1$$

26. $\cos 45^\circ \mid -\cos 60^\circ \mid \sqrt{2}/2$

6 marks

Explanation:

Given the following expression to simplify, without the use of a calculator:

$$\frac{\cos 315^\circ \cdot \sin 135^\circ}{\cos 240^\circ} \quad \text{Apply reduction formulae}$$

$$= \frac{\cos(360^\circ - 45^\circ) \cdot \sin(180^\circ - 45^\circ)}{\cos(180^\circ + 60^\circ)}$$

$$= \frac{\cos 45^\circ \cdot \sin 45^\circ}{-\cos 60^\circ} \quad \text{These are special angles}$$

$$= \frac{\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}{-\frac{1}{2}} \quad \text{Substitute ratios}$$

$$= -1$$

$$A = \cos 45^\circ$$

$$B = -\cos 60^\circ$$

$$C = \frac{\sqrt{2}}{2}$$

27. x^6

3 marks

Explanation:

$$\sqrt[3]{x^7} \cdot \sqrt[3]{x^{11}} \quad \text{Apply the surd law } \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$= \sqrt[3]{x^7 \cdot x^{11}} \quad \text{Simplify under surd}$$

$$= \sqrt[3]{x^{18}} \quad \text{Convert to exponential form}$$

$$= x^{\frac{18}{3}}$$

$$= x^6$$

28. C: $x = -2$ or $x = 0$

4 marks

Explanation:

$$3^{2x+2} + 1 = 10.3^x$$

Expand and equate to zero:

$$3^{2x}.3^2 - 10.3^x + 1 = 0$$

The exponents with x are in a 2 : 1 ratio.

Let $3^x = k$ and solve as a quadratic equation:

$$9k^2 - 10k + 1 = 0$$

$$(9k - 1)(k - 1) = 0$$

$$k = \frac{1}{9} \text{ or } k = 1$$

Now solve for x :

$$3^x = \frac{1}{9} \quad \text{or} \quad 3^x = 1$$

$$\begin{array}{ll} 3^x = 3^{-2} & 3^x = 3^0 \\ x = -2 & x = 0 \end{array}$$

29. 0 | 180 | 360

6 marks

Explanation:

Solve for θ , if $\theta \in (180^\circ; 360^\circ]$:

$$\sin \theta \cos \theta - \sin \theta = 0$$

$$\sin \theta (\cos \theta - 1) = 0$$

$$\sin \theta = 0 \text{ or } \cos \theta = 1$$

The solution for $\sin \theta = 0$ and $\cos \theta = 1$ is 0° . Using the general solution for $\sin \theta = 0$, the second angle will be 180° .

$$\theta = 0^\circ + k.360^\circ; k \in \mathbb{Z}$$

or

$$\theta = 180^\circ + k.360^\circ$$

$\theta \in (180^\circ; 360^\circ]$ means exclude 180° and include 360° . Therefore using the general solution, $\theta = 360^\circ$.

30. false

2 marks

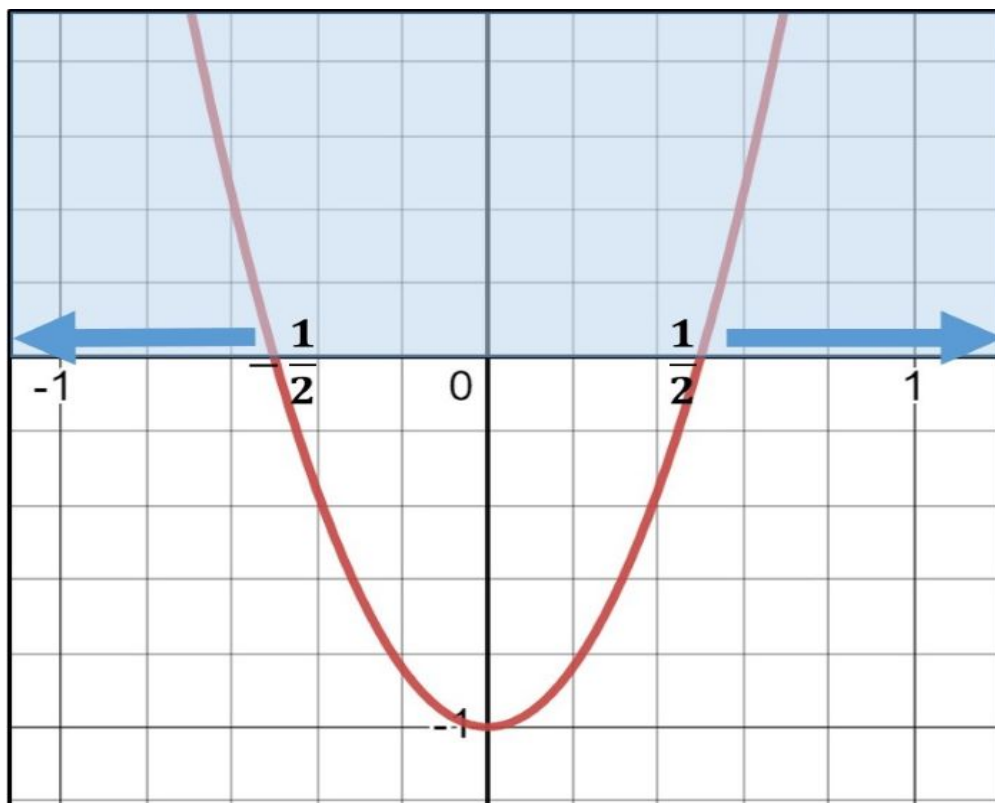
Explanation:

$$4x^2 - 1 > 0$$

$$(2x + 1)(2x - 1) > 0$$

The **roots of the function** is $x = -\frac{1}{2}$ or $x = \frac{1}{2}$.

The function has **a minimum** since $a > 0$.



The function is **greater than zero**, so the x -values that is a solution to this, is part of the shaded part where $y > 0$. Remember that $y = 0$ is not part of the solution.

The solution is $x < -\frac{1}{2}$ or $x > \frac{1}{2}$.

Total: 111 Marks